the number density and temperature of a hot inhomogeneous plasma. This could be done by measuring the phase and the amplitude of the wave as a function of z.

Appendix

Derivation of the Wave Equation in a Hot Inhomogeneous Stationary Plasma

Assume all quantities are of the form A_0+A_1 where A_0 indicates time-independent quantities and A_1 indicates time-varying quantities. Further assume that the ions of density N_0 are smeared out and stationary to form a neutralizing background. The instantaneous charge density

$$\rho = -(N_0+N_1)e + N_0 e = -N_1 e \quad (14)$$

and current

$$\bar{J} = -(N_0+N_1)e(\bar{v}_1) = -N_0 e\bar{v}_1 \quad (15)$$

together with Maxwell's equations

$$\frac{\partial \epsilon_0 \bar{E}}{\partial t} + \bar{J} = \operatorname{curl}\frac{\bar{B}}{\mu_0} \quad (16)$$

$$\frac{\partial \bar{B}}{\partial t} + \operatorname{curl}\bar{E} = 0 \quad (17)$$

$$\operatorname{div}\epsilon_0\bar{E} = \rho \quad (18)$$

lead to expressions (with assumed time dependence $\epsilon^{i\omega t}$)

$$+N_0 e\bar{v}_1 = \frac{i}{\omega\mu_0}\left(\frac{\omega^2}{c^2}\bar{E}_1 - \operatorname{curl}\operatorname{curl}\bar{E}_1\right) \quad (19)$$

$$N_1 = -\frac{\epsilon_0}{e}\operatorname{div}\bar{E}_1. \quad (20)$$

The linearized equation of motion (1) yields two terms

$$-eN_0\bar{E}_0 - \operatorname{grad}P_0 = 0 \quad (21)$$

$$i\omega m N_0\bar{v}_1 = -eN_1\bar{E}_0 - eN_0\bar{E}_1 - \operatorname{grad}P_1. \quad (22)$$

Combining (19) to (22) together with an assumed equation of state $p=\gamma NkT$, one obtains the wave equation (2).

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Comments on Excitation of Spin Waves by Wire Arrays

Messrs. LaRosa and Vasile[1]

Kaufman and Soohoo[2] have suggested that spin waves involving exchange forces can be excited by means of a fine wire at the end of a YIG crystal. It would be very desirable to do this, for then the entire crystal might be kept at a low enough dc field to avoid coupling to acoustic waves. We have made a detailed analysis of an array of flat wires immersed in an infinite YIG medium and also printed on the air-YIG interface of a semi-infinite YIG medium. The dc magnetic field has been taken both perpendicular to the plane of the array and parallel to the array conductors.

One idea which runs through the literature on the subject is that the rapid decay of the exciting field in the desired propagation direction should enable some net coupling to be obtained to the very short wavelength (10^{-5} cm) exchange spin waves. Accordingly, the currents in the adjacent wires were assumed to be oppositely directed. The assumption of an infinite array enabled the boundary value problem to be solved in rectangular coordinates. The magnetic field and the RF magnetization component could be expressed in terms of Fourier sine and cosine series as functions of the coordinate along the array (perpendicular to the conductors). No variation along the conductors was assumed. The variation perpendicular to the array has several propagation constant values given by a dispersion relation for each orientation of the dc field.

For the dc field perpendicular to the array, four values of wave vector k were found.

1) Low-k (electromagnetic). Decaying in propagation direction.
2) Medium-k propagating. Similar to magnetostatic waves (group velocity opposite to phase velocity).
3) High-k propagating. Involving exchange forces, obeying $\omega/\gamma = H_i + H_{ex}a^2k^2$.
4) High-k, nonpropagating. Circular polarization sense opposite to the high-k propagating. Obeying dispersion relation $-\omega/\gamma = H_i + H_{ex}a^2k^2$ (k^2 negative).

It was found that for all conductor widths and spacings, only low-k waves were excited. In fact, the ability of the low-k waves to satisfy the boundary conditions is enhanced by decreasing the width and spacing of the conductors. The plausibility argument for this is that the electromagnetic field of the conductors without any YIG obeys very closely Laplace's equation, i.e., $k\approx 0$. Therefore, there is no variation in the exciting field which tends to displace the spins against the exchange forces. Very close conductor spacing creates a fast decay perpendicular to the array and an equally fast periodic variation along the array. The second derivatives in the two directions are equal and opposite and the Laplacian is zero.

The situation is slightly different with the dc field parallel to the wires. The closeness of the conductors drops out of the dispersion relation. Three values of k are permitted.

1) Low-k (electromagnetic) becomes high-k nonpropagating as the field is increased.
2) High-k propagating becomes low-k as the field is increased.
3) High-k nonpropagating remains about the same.

There is a restricted range of field in which 1) and 2) become comparable, so that true spin waves involving exchange forces are excited. However, good excitation of propagating spin waves occurs only for spin wavelengths comparable to or greater than elastic wavelengths. Also, this region is very narrow band.

The transmission line analog used by Kaufman and Soohoo[3] assumes single mode propagation and reduces a two- or three-dimensional problem to a one-dimensional problem. The actual magnetic field of the conductor is replaced by sources distributed along the transmission line. This distributed source analog is based on Schlömann,[4] equation (10), which we believe to be a misinterpretation, as follows.

The equations of motion give the susceptibility tensor which relates the RF magnetization to the RF magnetic field in a gyrotropic medium. This tensor expresses the effects of forces imposed by the local magnetic field and the exchange forces on the spin dipole moments. These forces cause precession at an amplitude and rate consistent with the magnetogyric ratio.

When the susceptibility tensor is inserted in Maxwell's equations, a set of homogeneous equations can be obtained for either the magnetization or the magnetic field. The dispersion relation is obtained by setting the determinant equal to zero.

There are no source terms involved in the interior of the region. Amplitudes are determined by matching solutions at boundaries.

[1] Manuscript received November 8, 1965.
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The above procedure is used by Auld[5] in the solution for magnetostatic modes of spheres and is fully described in Lax and Button.[6] In Schlömann's (10), the magnetic field h is not a source term. It must have the same spatial variation as α, since both h and α are simply the variables in a homogeneous set of equations. If h has an exponential decay in the z-direction, so will α. That is, the solution will consist entirely of the low-k mode. If there is to be any high-k propagating spin wave in the solution for α, there must be a high-k propagating wave in h. Obviously, the unperturbed field of a conductor will not contain a short-wavelength variation. The boundary value problem must be solved to determine whether such a component is present in the YIG.

We believe that Schlömann's (12) is not an inhomogeneous equation and that the distributed source interpretation is not correct. The calculations which follow his (12) are, however, essentially correct because the excitation is not calculated from the unperturbed (without YIG) magnetic field.

Incorrect interpretation of Schlömann's (10) leads to serious errors when the frequency is such that short wavelength spin waves are allowed by the dispersion relation. Under these conditions, the low-k (electromagnetic) solution is also allowed and the h term in Schlömann's (10) matches the unperturbed field almost exactly. ("Unperturbed" means YIG dielectric constant assumed but magnetic dipoles ignored.) Our boundary value solutions show that very little spin wave excitation is obtained, and we think that it makes little difference how many conductors are used or whether they are flat or round.

To date, the only effective linear method of coupling an electromagnetic field to high-k, propagating spin waves is, as predicted by Schlömann, to couple at a point where the spin-wavelength approaches the electromagnetic wavelength. By shaping the dc magnetic field, one can slide along the dispersion relation to the high-k region. Under these conditions, spin-phonon coupling is unavoidable and spin wave defocusing can become a serious problem.

Our calculations are lengthy and not suitable for this published correspondence. They will be supplied to interested workers.

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Messrs. Kaufman and Soohoo[7]

The idea of exciting spin waves in a material by excitation of a specimen with a field that extends only a short distance into the material was proposed before us by Lüthi.[8] In our paper[3] we describe a possible method of accomplishing this by using a fine wire. LaRosa and Vasile maintain that this scheme (and, by inference, any similar scheme) is not valid, since the magnetization must always have the same periodicity as the applied RF h-field; and that the RF h-field must have the same spatial variation as the magnetization.

It should be pointed out here that the excitation of a standing spin wave in a film, first demonstrated by Seavey and Tannenwald,[9] is accomplished with an RF magnetic field distribution that is essentially of constant amplitude throughout the film, yet the spin wave local amplitude varies in the manner of a standing wave distribution. Accordingly, it is not necessary to have the same spatial distribution for RF field as for magnetization.

LaRosa and Vasile state that Schlömann's (12)[4] is not an inhomogeneous equation. This equation is based on the equation of motion of the magnetization,

$$\frac{\partial \boldsymbol{M}}{\partial t} = \gamma(\boldsymbol{M} \times \boldsymbol{H}) + \gamma H_{ex} a^2 \frac{\boldsymbol{M} \times \nabla^2 \boldsymbol{M}}{|\boldsymbol{M}|}. \quad (1)$$

For the infinite medium, assuming fields $\boldsymbol{H} = \hat{a}_x h_x + \hat{a}_y h_y + \hat{a}_z H_0$ and magnetization $\boldsymbol{M} = \hat{a}_x m_x + \hat{a}_y m_y + \hat{a}_z M_0$, where all RF terms vary as exp ($j\omega t$), this equation is

$$\frac{\partial^2 m_0}{\partial z^2} + \left(\frac{-\gamma H_0 - \omega}{\gamma H_{ex} a^2}\right) m_0 = -\frac{M_0}{H_{ex} a^2} h_0. \quad (2)$$

Here, following Schlömann,[4] $h_0 = h_x + jh_y$; $m_0 = m_x + jm_y$. Neither the spatial distribution of h_0 nor that of m_0 have been specified. As usual, the solution to the homogeneous portion of this equation results in $m_0 = A \exp(jkz) + B \exp(-jkz)$, where $k = [(-\gamma H_0 - \omega)/\gamma H_{ex} a^2]^{1/2}$ is the usual dispersion relation for z-directed spin waves. (Here only variation with z was assumed.) Equation (2) above is therefore clearly inhomogeneous; the h_0 allowing for additional solutions. In these, the A and B coefficients are now not constants where h_0 is nonzero, but they allow for "normal mode" propagation of A, B where $h_0 = 0$. The scheme is analogous to the piezoelectric transducer.

Since this analysis is based solely on the equation of motion (1), it is not completely correct; for a more rigorous solution would also include Maxwell's equations. However, since the exchange power content of a spin wave far exceeds that of the usual Poynting vector power,[10] this treatment should be fairly accurate.

It is of interest to mention here that an earlier analysis[11] states the relation between $\boldsymbol{h}_k$ and $\boldsymbol{m}_k$ of a spin wave, based on Maxwell's equations. In this normal mode analysis, we find

$$\boldsymbol{h}_k = \frac{4\pi(\omega^2 \epsilon_e / c^2)\boldsymbol{m}_k - 4\pi \boldsymbol{k}(\boldsymbol{k} \cdot \boldsymbol{m}_k)}{k^2 - \omega^2 \epsilon_e / c^2}. \quad (3)$$

Curiously, $\boldsymbol{h}_k$ and $\boldsymbol{m}_k$ here do not have the same spatial distribution, because of the presence of the $\boldsymbol{k} \cdot \boldsymbol{m}_k$ term.

In conclusion, we wish to state that while our transmission line one-dimensional analysis is a simplification over the actual physical picture, we feel that this analysis is still correct in principle, in the light of the arguments presented here.

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Messrs. LaRosa and Vasile[12]

1) Our comments apply to homogeneous magnetic insulators. Spin waves are generated in thin films because of one or more of the following complications: finite conductivity, variation of $4\pi M_s$ with depth, surface pinning (which prevents uniform precession), and small sample size in direction of propagation.

2) We are not sufficiently familiar with the piezoelectric transducer analogy to judge its relevance.

3) We still believe that Maxwell's equations solved simultaneously with the equations of motion yield a homogeneous set of equations in $\bar{m}$, $\bar{h}$ or $\bar{e}$. There are no sources distributed in the volume of the YIG. There are no perturbations in the YIG medium which could serve as sources via perturbation theory.

[12] Manuscript received March 8, 1966.

Reflection Measurements with Broadband Frequency Modulation Using Long Transmission Lines

This correspondence describes some applications of broadband frequency modulation for measuring reflections on moderately long transmission lines. As known from earlier publications on this subject [1], [2], and from FM-radar techniques, a frequency modulated wave train from a sweep generator is fed into a transmission line. A part of the energy is scattered back by reflections produced on the line or at the end of it. A detector conveniently coupled to the line near the generator provides for mixing of transmitted and scattered wave amplitudes, thus generating an intermediate frequency signal (generally 0.1 . . . 15 kHz) which can be processed by audio frequency techniques.

Ideally, the following equation holds for this audio signal:

$$V_d = \sum_n k r_n \cos\left(\frac{4\pi d_n f_m \Delta F}{v} t - \phi_n\right) \quad (1)$$

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